Battery Outcomes Summary for the Mena Dominance Law

Experimental Battery Runs, Step-Down Schedules, Outcomes, and BCM Reference Logic

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Abstract

This document provides a battery-only outcomes summary derived from the executed battery section of the Mena Dominance Law paper. It is intended as a downloadable companion file for website users who want a clean summary of the battery results without the rest of the paper.

The document reports: (i) the battery models and usable run counts, (ii) the fixed four-step Mena step-down schedules used near cutoff, (iii) per-battery outcomes comparing the regular constant-current baseline (R) versus the Mena viability-governed policy (M), (iv) pooled outcomes across all packs, (v) paired complete-case recovery results, and (vi) a reference BCM-oriented control implementation showing how the paper's logic can be implemented in software for discharge governance.

CSV files are provided separately for users who want the data tables and valid raw run files. Invalid files are excluded.

Keywords: battery discharge, viability-governed control, step-down current, delivered energy, delivered charge, runtime, recovery voltage, BCM logic

Contents

1 Purpose of this Download

This download is a battery-only summary package derived from the battery experiments reported in the Mena Dominance Law paper. It is intended for website users who want the battery results in a compact form.

The package is structured to answer the main practical questions:

- Which batteries were tested?
- How many valid runs were used?
- What step-down schedule was used for each pack?
- What did each battery achieve under M versus R?
- What were the pooled results across all batteries?
- What would the control logic look like in a BCM-style implementation?

2 Battery Models and Usable Dataset

Four physical packs were tested:

- **DCB205**: DeWalt 20V MAX 5Ah pack
- **KB224-03**: Kobalt 24V MAX 2Ah pack
- **R840040_A**: Ridgid 18V 4Ah pack (Battery A)
- **R840040_B**: Ridgid 18V 4Ah pack (Battery B)

The final usable dataset is balanced:

$$N_M = 57, \qquad N_R = 57.$$

Pack-level run counts are:

- DCB205: 15/15
- KB224-03: 14/14
- R840040_A: 14/14
- R840040_B: 14/14

A small number of originally recorded files were flagged invalid and replaced by reruns. Invalid files are excluded from this package.

Table 1: Battery packs and valid run counts (balanced usable set).

Pack	**Pack identity**	**Runs (M/R)**
DCB205	DeWalt 20V MAX 5Ah	15/15
KB224-03	Kobalt 24V MAX 2Ah	14/14
R840040_A	Ridgid 18V 4Ah (Battery A)	14/14
R840040_B	Ridgid 18V 4Ah (Battery B)	14/14
ALL	Pooled	57/57

3 Policies Compared

Two discharge policies were compared:

- **R (regular baseline):** constant-current discharge, terminated at first detection of pack voltage at or below the fixed pack-specific cutoff.
- **M (Mena viability governor):** a four-step monotone current reduction schedule near cutoff, implemented on the ET5406A+ Battery-Test CC interface.

The battery embodiment uses the terminal voltage margin to fixed cutoff as its real-time viability observable:

$$\widehat{\Delta}_{\text{batt}}(t) = V(t) - V_{\text{cut}}.$$

As the pack approaches the cutoff boundary, the M policy reduces current to avoid sustained near-boundary exposure while preserving the same fixed cutoff rule.

4 Hardware and Test Setup

All battery discharge runs were executed using:

- **Electronic load:** Yertai ET5406A+
- **Mode:** constant-current and Battery-Test CC multi-step mode
- **Protection:** inline 15 A fuse on the positive lead
- **Charging:** original manufacturer chargers only
- **Signals logged:** pack voltage $V(t)$, current $I(t)$, and optional temperature $T(t)$
- **Cadence:** typical logger cadence of approximately 2 s
- **Termination rule:** stop at first detection of $V_{\text{pack}}(t) \leq V_{\text{cut}}$
- **Recovery endpoint:** standardized 60 s post-discharge recovery, when available

Ambient temperature was not regulated; experiments were executed across approximately 68°F to 100°F. Rest protocol:

- Pre-run rest ≥ 1 hour after use
- Post-run rest ≥ 1 hour after test before charge or re-run

5 Step-Down Schedules Used

The M schedules were pack-specific fixed four-step discharge profiles with a common floor:

$$I_4 = 2.0 \text{ A}.$$

The generic form is:

$$I(t) = \begin{cases} I_1, & V(t) > V_1, \\ I_2, & V_1 \geq V(t) > V_2, \\ I_3, & V_2 \geq V(t) > V_3, \\ I_4, & V_3 \geq V(t) > V_{\text{cut}}, \\ 0, & V(t) \leq V_{\text{cut}}. \end{cases} \qquad I_1 > I_2 > I_3 > I_4.$$

Table 2: Pack-specific M schedules used in the study (ET5406A+ BATT-CC; four-step drop-down with $I_4 = 2.0$ A floor).

Pack	**M schedule (voltage window → current)**
DCB205 (5S)	$V > 18.00$: 7.50 A; $18.00 \geq V > 17.00$: 5.00 A; $17.00 \geq V > 16.50$: 3.00 A; $16.50 \geq V > 16.00$: 2.00 A; Stop at $V \leq 16.00$ V.
KB224-03 (6S)	$V > 21.60$: 7.50 A; $21.60 \geq V > 20.40$: 5.00 A; $20.40 \geq V > 19.80$: 3.00 A; $19.80 \geq V > 19.20$: 2.00 A; Stop at $V \leq 19.20$ V.
R840040_A (5S)	$V > 18.00$: 7.50 A; $18.00 \geq V > 17.00$: 5.00 A; $17.00 \geq V > 16.50$: 3.00 A; $16.50 \geq V > 16.00$: 2.00 A; Stop at $V \leq 16.00$ V.
R840040_B (5S)	$V > 18.00$: 7.50 A; $18.00 \geq V > 17.00$: 5.00 A; $17.00 \geq V > 16.50$: 3.00 A; $16.50 \geq V > 16.00$: 2.00 A; Stop at $V \leq 16.00$ V.

6 Endpoints Reported

Primary endpoints:

- runtime to cutoff t_{cut}
- delivered energy to cutoff

$$Wh_{\text{to cut}} = \int_0^{t_{\text{cut}}} V(t)|I(t)|dt$$

- delivered charge to cutoff

$$Ah_{\text{to cut}} = \int_0^{t_{\text{cut}}} |I(t)|dt$$

- initial sag η_0

Secondary endpoints (paired complete-case):

- recovery voltage at 60 s
$$V_{\text{rec}} \equiv V(t_{\text{cut}} + 60\text{ s})$$
- first recovery sample after load removal $V_{\text{rec},0}$
- recovery rise
$$\Delta V_{\text{rec}} = V_{\text{rec}} - V_{\text{rec},0}$$

7 Pooled Battery Outcomes

Across all four packs, the pooled medians show that the M policy outperformed the regular constant-current baseline under identical cutoff rules.

Table 3: Primary and secondary outcomes (pooled; medians). Primary endpoints use all runs. Secondary endpoints use paired complete-case inclusion.

Endpoint	**M median**	**R median**	**Δ (M–R)**	**% vs R**
Runtime t_{cut} (s)	2869	1756	+1113	+63.38%
Delivered energy $Wh_{\text{to cut}}$ (Wh)	68.14	64.39	+3.75	+5.82%
Delivered charge $Ah_{\text{to cut}}$ (Ah)	3.825	3.650	+0.175	+4.80%
Initial sag η_0 (V)	0.928	0.968	−0.040	−4.13%
Temperature pairs: $n = 57$ paired complete-case. Median $(T_{\max,M} - T_{\max,R}) = -0.75$°C; approximately 62% cooler under M (descriptive only).				
Recovery pairs: $n = 54$ paired complete-case. Median recovery-rise diff ≈ -0.137 V (M lower). Median end-recovery diff ≈ -0.568 V (M lower).				

8 Per-Battery Outcomes

Each pack showed an increase in delivered charge and delivered energy under the M policy relative to the R baseline. Runtime increased in all packs, which is expected because the M policy reduces current near cutoff. Initial sag moved in a favorable direction for three of the four packs.

Table 4: Per-pack primary endpoint medians (M vs. R) with median difference and percent change relative to R.

Pack	Endpoint	M median	R median	Δ (M–R)	% vs R
DCB205	Delivered charge $Ah_{\text{to cut}}$ (Ah)	4.299	4.139	+0.159	+3.85%
DCB205	Delivered energy $Wh_{\text{to cut}}$ (Wh)	77.65	74.49	+3.17	+4.25%
DCB205	Initial sag η_0 (V)	0.599	0.604	-0.005	-0.83%
DCB205	Runtime t_{cut} (s)	3134	2050	+1084	+52.88%
KB224-03	Delivered charge $Ah_{\text{to cut}}$ (Ah)	2.195	2.061	+0.134	+6.52%
KB224-03	Delivered energy $Wh_{\text{to cut}}$ (Wh)	50.03	46.91	+3.12	+6.65%
KB224-03	Initial sag η_0 (V)	0.979	0.971	+0.008	+0.85%
KB224-03	Runtime t_{cut} (s)	2794	1566	+1228	+78.41%
R840040_A	Delivered charge $Ah_{\text{to cut}}$ (Ah)	3.447	3.268	+0.179	+5.48%
R840040_A	Delivered energy $Wh_{\text{to cut}}$ (Wh)	60.74	57.77	+2.96	+5.13%
R840040_A	Initial sag η_0 (V)	1.312	1.349	-0.037	-2.76%
R840040_A	Runtime t_{cut} (s)	2918	1755	+1163	+66.26%
R840040_B	Delivered charge $Ah_{\text{to cut}}$ (Ah)	3.827	3.651	+0.175	+4.80%
R840040_B	Delivered energy $Wh_{\text{to cut}}$ (Wh)	68.44	64.48	+3.96	+6.14%
R840040_B	Initial sag η_0 (V)	0.868	0.938	-0.070	-7.43%
R840040_B	Runtime t_{cut} (s)	2918	1713	+1205	+70.35%

9 Recovery Outcomes by Battery

Recovery metrics were computed on paired complete-case runs only. Lower standardized recovery voltages under M are interpreted here as a boundary-adjacent recovery signature consistent with longer or deeper operation under fixed cutoff rules, not as direct degradation evidence.

Table 5: Recovery outcomes by pack (paired complete-case). $V_{\text{rec},0}$ is the first recorded sample after load removal; V_{rec} is the nearest sample to $t_{\text{cut}} + 60$ s; $\Delta V_{\text{rec}} = V_{\text{rec}} - V_{\text{rec},0}$. Medians are reported; Δ columns are (M–R).

Pack	n	$V_{\text{rec},0}$ M	$V_{\text{rec},0}$ R	Δ	V_{rec} M	V_{rec} R	Δ	ΔV_{rec} M	ΔV_{rec} R	Δ
DCB205	13	16.170	16.580	-0.410	16.278	16.849	-0.571	0.115	0.279	-0.164
KB224-03	14	19.870	20.390	-0.520	20.343	20.794	-0.451	0.463	0.392	+0.072
R840040_A	13	16.240	16.710	-0.470	16.472	17.096	-0.624	0.220	0.359	-0.139
R840040_B	14	16.130	16.620	-0.490	16.392	16.977	-0.585	0.258	0.362	-0.104
Pooled	54	16.190	16.640	-0.450	16.409	16.977	-0.568	0.225	0.362	-0.137

10 Interpretation of the Battery Results

Under identical cutoff boundaries, the M policy increased delivered work to cutoff:

- more delivered energy
- more delivered charge
- longer runtime
- lower pooled initial sag

The runtime gain is policy-mediated because M intentionally lowers current near cutoff, so runtime should not be treated as a standalone efficiency claim. The main cross-policy outcomes are the gains in delivered energy and delivered charge under the same fixed boundary definitions.

These differences are not a bookkeeping artifact, because:

- both policies used the same pack-specific cutoff
- both policies terminated at first detection of $V(t) \leq V_{\text{cut}}$
- energy and charge were computed from the same measured voltage and current signals on the active discharge segment

11 CSV Files Included in the Download Package

The website download package can include these CSV files:

- `reported_overall_outcomes.csv`
- `reported_per_battery_outcomes.csv`
- `reported_recovery_outcomes.csv`
- `reported_stepdown_schedules.csv`
- `valid_run_index.csv`
- valid raw run CSV files only

Invalid files should remain excluded. Users who want the summary can read this PDF; users who want the run-level files can open the CSV package.

12 BCM-Oriented Reference Logic

The paper's battery controller logic can be written as a continuous software governor rather than only as a fixed ET5406A+ step profile. The software idea is:

1. estimate a near-rest voltage V_{rest}
2. estimate an effective use-resistance $\hat{R}_{\text{use}}$

3. compute an admissible current ceiling

$$I_{\max}(t) \approx \frac{V_{\text{rest}}(t) - V_{\text{cut}}}{\hat{R}_{\text{use}}(t)}$$

4. clip requested discharge current to that ceiling

This gives a BCM-style real-time discharge governor that preserves the same cutoff rule while reducing demand near the boundary.

12.1 Reference BCM discharge governor pseudocode

Algorithm 1: Continuous battery viability governor for BCM-oriented discharge control.

```
Algorithm: BCM_Battery_Viability_Discharge_Governor

Inputs:
  V_term[k] # measured pack terminal voltage
  I_meas[k] # measured current
  I_demand[k] # requested discharge current
  V_cut # fixed cutoff voltage

Parameters:
  V_open_init # initial near-rest voltage estimate
  R_init # initial use-resistance estimate
  R_min # minimum admissible resistance
  I_rest_max # current threshold for near-rest update
  I_R_min # minimum current for resistance update
  alpha_V # smoothing factor for V_rest_hat
  alpha_R # smoothing factor for R_use_hat
  I_hw_max # hardware current ceiling
  tau_r_batt # optional violation threshold

Initialization:
  V_rest_hat = V_open_init
  R_use_hat = R_init

For each sample k:
  V = V_term[k]
  I = I_meas[k]
  I_req = max(0, I_demand[k])

  if abs(I) <= I_rest_max:
      V_rest_hat = (1 - alpha_V) * V_rest_hat + alpha_V * V

  I_mag = max(abs(I), I_R_min)
  sag = V_rest_hat - V

  if (abs(I) >= I_R_min) and (sag > 0):
      R_sample = sag / I_mag
      R_use_hat = (1 - alpha_R) * R_use_hat + alpha_R * R_sample

  R_eff = max(R_use_hat, R_min)
  headroom = V_rest_hat - V_cut

  I_max = headroom / R_eff
  I_max = clip(I_max, 0.0, I_hw_max)
```

```
  I_cmd = min(I_req, I_max)

  delta = V - V_cut
  violation = 1 if (delta <= tau_r_batt) else 0

Return:
  I_cmd, delta, violation
```

12.2 Reference fixed step-down implementation for embedded use

If a simpler embedded implementation is preferred, the BCM can use the same fixed profile logic as the ET5406A+ implementation:

Algorithm 2: Pack-specific fixed step-down logic for embedded discharge control.

```
Algorithm: BCM_Fixed_Stepdown_Discharge

Inputs:
  V_pack
  pack_id

If pack_id == DCB205 or R840040_A or R840040_B:
    if V_pack > 18.00: I_cmd = 7.50
    else if V_pack > 17.00: I_cmd = 5.00
    else if V_pack > 16.50: I_cmd = 3.00
    else if V_pack > 16.00: I_cmd = 2.00
    else: I_cmd = 0.00

If pack_id == KB224_03:
    if V_pack > 21.60: I_cmd = 7.50
    else if V_pack > 20.40: I_cmd = 5.00
    else if V_pack > 19.80: I_cmd = 3.00
    else if V_pack > 19.20: I_cmd = 2.00
    else: I_cmd = 0.00

Return I_cmd
```

13 Charging Notes

The battery experiments in the paper did **not** introduce a custom charging algorithm. All packs were charged with original manufacturer chargers and no modified charge-control law was claimed.

So for a BCM-oriented summary, the correct statement is:

- **Discharge control logic:** explicitly defined by the Mena viability-governed discharge policy
- **Charging control logic:** not modified in this study; OEM charging retained

If you want to show that in code form:

Algorithm 3: Charging policy used in the reported study.

```
Algorithm: Charging_Policy_Used_In_Study

For each battery pack:
    connect pack to original manufacturer charger
```

```
    allow standard OEM charging process
    do not apply custom charging law
    after charging, enforce required rest period before testing

Return fully charged pack for next run
```

14 Replication Note

For website users who want to reproduce the battery outcome package, the minimum required reporting set is:

- pack ID
- policy ID (M or R)
- cutoff threshold
- schedule used
- measured voltage and current trace
- runtime to cutoff
- integrated charge and energy to cutoff
- initial sag
- recovery data when available

15 Bottom Line

The battery results show that under the same fixed cutoff rules, the Mena viability-governed step-down discharge policy delivered more usable work to cutoff than the regular constant-current baseline.

Pooled medians show:

- **Energy:** +5.82%
- **Charge:** +4.80%
- **Runtime:** +63.38%
- **Initial sag:** −4.13%

That is the core battery outcome the website download should communicate.